example: $a(b+c)$ means a times $(b+c)$

Problems 1-4: What is the meaning of:

1. abc
2. $2a$
3. c^2
4. $3(a-4)$

To show *division*, fractions are often used:
example: 3 divided by 6 may be shown: $3 \div 6$ or $\frac{3}{6}$, or $6\overline{)3}$, and all have value $\frac{1}{2}$, or .5.

5. What does $\frac{4a}{3b}$ mean?

Problems 6-15: Write a fraction form and reduce to find the value (if possible):

6. $36 \div 9 =$
7. $4 \div 36 =$
8. $10\overline{)36} =$
9. $1.2\overline{).06} =$
10. $2x \div a =$
11. $12 \div 5y =$
12. $6b \div 2a =$
13. $8r \div 10s =$
14. $a \div a =$
15. $2x \div x =$

In the above exercises, notice that the fraction forms can be reduced if there is a common (shared) factor in the top and bottom:

example: $\frac{36}{9} = \frac{9 \bullet 4}{9 \bullet 1} = \frac{9}{9} \bullet \frac{4}{1} = 1 \bullet 4 = 4$

example: $\frac{4}{36} = \frac{4 \bullet 1}{4 \bullet 9} = \frac{4}{4} \bullet \frac{1}{9} = 1 \bullet \frac{1}{9} = \frac{1}{9}$

example: $\frac{36}{10} = \frac{2 \bullet 18}{2 \bullet 5} = \frac{18}{5}$ *(or* $3\frac{3}{5}$*)*

example: $\frac{6b}{2a} = \frac{2 \bullet 3 \bullet b}{2 \bullet a} = \frac{3b}{a}$

example: $\frac{a}{a} = 1$

example: $\frac{2x}{x} = \frac{2 \bullet x}{1 \bullet x} = \frac{2}{1} \bullet \frac{x}{x} = \frac{2}{1} \bullet 1 = 2 \bullet 1 = 2$

Problems 16-24: Reduce and simplify:

16. $\frac{3x}{3} =$
17. $\frac{3x}{4x} =$
18. $\frac{x}{2x} =$
19. $\frac{12x}{3x} =$
20. $\frac{12x}{3} =$
21. $\frac{abc}{3ac} =$
22. $\frac{6x}{8xy} =$
23. $\frac{15x^2}{10x} =$
24. $\frac{6x}{5} \bullet 5 =$

(Hint: $\frac{6x}{5} \bullet 5 = \frac{6x}{5} \bullet \frac{5}{1} = \frac{6x \bullet 5}{5 \bullet 1}$ ...)

The *distributive property* says $a(b+c) = ab + ac$. Since equality $(=)$ goes both ways, the distributive property can also be written $ab + ac = a(b+c)$. Another form it often takes is $(a+b)c = ac + bc$, or $ac + bc = (a+b)c$.

example: $3(x-y) = 3x - 3y$. Comparing this with $a(b+c) = ab + ac$, we see $a = 3$, $b = x$, and $c = -y$

example: Compare $4x + 7x = (4+7)x = 11x$ with $ac + bc = (a+b)c$; $a = 4,\ c = x,\ b = 7$
example: $4(2+3) = 4 \bullet 2 + 4 \bullet 3$ (The distributive property says this has value 20, whether you do $4 \bullet 5$ or $8 + 12$.)
example: $4a + 6x - 2 = 2(2a + 3x - 1)$

Problems 25-35: Rewrite, using the distributive property:

25. $6(x-3) =$
26. $4(b+2) =$
27. $(3-x)2 =$
28. $4b - 8c =$
29. $4x - x =$ (Hint: think $4x - 1x$, or 4 cookies minus 1 cookie)
30. $-5(a-1) =$
31. $5a + 7a =$
32. $3a - a =$
33. $3a - a =$
34. $5x - x + 3x =$
35. $x(x+2) =$

B. <u>Evaluation of an expression by substitution</u>:

example: Find the value of $7 - 4x$, if $x = 3$: $7 - 4x = 7 - 4 \bullet 3 = 7 - 12 = -5$
example: If $a = -7$ and $b = -1$, then $a^2b = (-7)^2(-1) = 49(-1) = -49$
example: If $x = -2$, then $3x^2 + x - 5$ $= 3(-2)^2 + (-2) - 5 = 3 \bullet 4 - 2 - 5$ $= 12 - 2 - 5 = 5$
example: $8 - c = 12$ Add c to each, giving $8 - c + c = 12 + c$, or $8 = 12 + c$. Then subtract 12, to get $-4 = c$, or $c = -4$.
example: $\frac{a}{5} = \frac{4}{3}$ Multiply each by the lowest common denominator 15: $\frac{a}{5} \bullet 15 = \frac{4}{3} \bullet 15$, or $a \bullet 3 = 4 \bullet 5$
Then divide by 3: $\frac{a \bullet 3}{3} = \frac{20}{3}$, so $a = \frac{20}{3}$

Problems 36-46: Given $x = -1, y = 3$ and $z = -3$, find the value:

36. $2x =$
37. $-z =$
38. $xz =$
39. $y + z =$
40. $z + 3 =$
41. $y^2 + z^2 =$
42. $2x + 4y =$
43. $2x^2 - x - 1 =$
44. $(x+z)^2 =$
45. $x^2 + z^2 =$
46. $-x^2 z =$

Problems 47-54: Find the value, given $a = -1,\ b = 2,\ c = 0,\ x = -3,\ y = 1$, and $z = 2$:

47. $\frac{6}{b} =$
48. $\frac{x}{a} =$
49. $\frac{x}{3} =$
50. $\frac{a-y}{b} =$
51. $\frac{4x-3y}{3y-2x} =$
52. $\frac{b}{c} =$
53. $-\frac{b}{z} =$
54. $\frac{c}{z} =$

C. Solving a linear equation in one variable:
Add or subtract the same thing on each side of the equation and/or multiply or divide each side by the same thing, with the goal of getting the variable alone on one side. If there are fractions, you can eliminate them by multiplying both sides of the equation by a common denominator. If the equation is a proportion, you may wish to cross-multiply.

> *example:* $3x = 10$ Divide both sides by 3, to get $1x$: $\frac{3x}{3} = \frac{10}{3}$, or $x = \frac{10}{3}$
> *example:* $5 + a = 3$ Subtract 5 from each side, to get $1a$ (which is *a*): $5 + a - 5 = 3 - 5$ or $a = -2$
> *example:* $\frac{y}{3} = 12$ Multiplying both sides by 3, to get y: $\frac{y}{3} \bullet 3 = 12 \bullet 3$, which gives $y = 36$.
> *example:* $b - 4 = 7$ Add 4, to get $1b$: $b - 4 + 4 = 7 + 4$, or $b = 11$

Problems 55-65: Solve:

55. $2x = 94$
56. $3 = \frac{6x}{5}$
57. $3x + 7 = 6$
58. $\frac{x}{3} = \frac{5}{4}$
59. $5 - x = 9$
60. $x = \frac{2x}{5} + 1$
61. $4x - 6 = x$
62. $x - 4 = \frac{x}{2} + 1$
63. $6 - 4x = x$
64. $7x - 5 = 2x + 10$
65. $4x + 5 = 3 - 2x$

Problems 66-70: Substitute the given value, then solve for the other variable:

> *example:* If $n = r + 3$ and $r = 5$ find the value of *n*: Replacing *r* with 5 gives $n = 5 + 3 = 8$.

66. $n = r + 3, \; n = 5$
67. $n = r + 3, \; n = 1$
68. $\frac{a}{2} = b, \; b = 6$
69. $5x = y - 3, \; x = 4$
70. $5x = y - 3, \; y = 3$

D. Word Problems:
If an object moves at a constant rate of speed *r*, the distance *d* it travels in time *t* is given by the formula $d = rt$.

> *example:* If $t = 5$ and $d = 50$, find *r*: Substitute the given values in $d = rt$ and solve: $50 = r \bullet 5$, giving $r = 10$.

Problems 71-72: In $d = rt$, substitute, then solve for the variable:

71. $t = 5, \; r = 50; \; d =$
72. $d = 50, \; r = 4; \; t =$

73. On a 40 mile hike, a strong walker goes 3 miles per hour. How much time will the person hike? Write an equation, then solve it.

74. "Product"
75. "Quotient"
76. "Difference"
77. "Sum"

78. The sum of two numbers is 43. One of the two numbers is 17. What is the other?
79. Write an equation which says that the sum of a number *n* and 17 is 43.
80. Write an equation which says the amount of simple interest *A* equals the product of the invested principle *P*, the rate of interest *r*, and the time *t*.
81. Use the equation of problem 80: $P = \$200, \; r = 7\%$ and $t = 5$ years. Find the amount of interest *A*.

Problems 82-83: In a rectangle which has two sides of length *a* and two sides of length *b*, the perimeter *P* is found by adding all the side lengths, or $P = 2a + 2b$.

82. If $a = 5$ and $b = 8$, find *P*.
83. If $a = 7$ and $P = 40$, find *b*.

Problems 84-85: The difference of two numbers *x* and 12 is 5.

84. If *x* is the *larger*, an equation, which says this same thing could be $x - 12 = 5$. Write an equation if x is the *smaller* of the two numbers *x* and 12.
85. Find the two possible values of *x* by solving each equation in problem 84.

Problems 86-87: Write an equation, which says:

86. *n* is 4 more than 3.
87. 4 less than *x* is 3.
88. Solve the two equations you wrote for problems 86 and 87.

Answers:

1. *a* times *b* times *c*
2. 2 times *a*
3. *c* times *c*
4. 3 times $(a - 4)$
5. $4a$ divided by $3b$
6. $\frac{36}{9} = 4$
7. $\frac{4}{36} = \frac{1}{9}$
8. $\frac{36}{10} = \frac{18}{5}$
9. $\frac{.06}{1.2} = \frac{6}{120} = \frac{1}{20}$
10. $\frac{2x}{a}$
11. $\frac{12}{5y}$

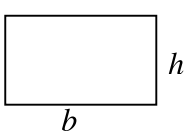
12. $\frac{6b}{2a}=\frac{3b}{a}$
13. $\frac{8r}{10s}=\frac{4r}{5s}$
14. $\frac{a}{a}=1$
15. $\frac{2x}{x}=2$
16. x
17. $\frac{3}{4}$
18. $\frac{1}{2}$
19. 4
20. $4x$
21. $\frac{b}{3}$
22. $\frac{3}{4y}$
23. $\frac{3x}{2}$
24. $6x$
25. $6x-18$
26. $4b+8$
27. $6-2x$
28. $4(b-2c)$
29. $(4-1)x=3x$
30. $-5a+5$
31. $(5+7)a=12a$
32. $(3-2)a=1a=a$
33. $2a$
34. $7x$
35. x^2+2x
36. –2
37. 3
38. 3
39. 0
40. 0
41. 18
42. 10
43. 2
44. 16
45. 10
46. 3
47. 3
48. 3
49. –1
50. –1
51. $-\frac{5}{3}$
52. no value (undefined)
53. –1
54. 0
55. $x=47$
56. $x=\frac{5}{2}$
57. $x=-\frac{1}{3}$
58. $x=\frac{15}{4}$
59. $x=-4$
60. $x=\frac{5}{3}$
61. $x=2$
62. $x=10$
63. $x=\frac{6}{5}$
64. $x=3$
65. $x=-\frac{1}{3}$
66. $5=r+3;\ r=2$
67. $1=r+3;\ r=-2$
68. $\frac{a}{2}=6;\ a=12$
69. $5\bullet 4=y-3; y=23$
70. $5x=3-3;\ x=0$
71. $d=50\bullet 5;\ d=250$
72. $50=4t;\ t=\frac{25}{2}$
73. $40=3t;\ t=\frac{40}{3}$ hours
74. Multiply
75. Divide
76. Subtract
77. Add
78. 26
79. $n+17=43$
80. $A=Prt$
81. $A=\$70$
82. $P=26$
83. $b=13$
84. $12-x=5$
85. $x=17$ or 7
86. $n=4+3$
87. $x-4=3$
88. $n=7;\ x=7$

TOPIC 6: GEOMETRY

A. <u>Formulas for perimeter *P* and area *A* of rectangles, squares, parallelograms, and triangles</u>:

Rectangle with base b and altitude (height) h:

$P=2b+2h$
$A=bh$

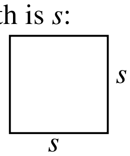


If a wire is bent in this shape, the perimeter P is the length of the wire, and the area A is the number of square units enclosed by the wire.

> *example:* A rectangle with $b=7$ and $h=8$:
> $P=2b+2h=2\bullet 7+2\bullet 8=14+16=30$ units
> $A=bh=7\bullet 8=56$ square units

A *square* is a rectangle with all sides equal, so the rectangle formulas apply (and simplify). If the side length is s:

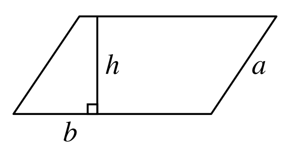
$P=4s$
$A=s^2$

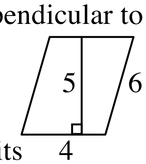


> *example:* A square with side $s=11$cm has
> $P=4s=4\times 11=44$ cm
> $A=s^2=11^2=121\,\text{cm}^2$ (sq. cm)

A *parallelogram* with base b and height h and other side a:

$A=bh$
$P=2a+2b$



> *example:* A parallelogram has sides 4 and 6; 5 is the length of the altitude perpendicular to the side 4.
> $P=2a+2b=2\bullet 6+2\bullet 4$
> $=12+8=20$ units
> $A=bh=4\bullet 5=20$ square units
>
> 
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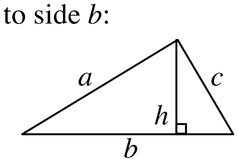
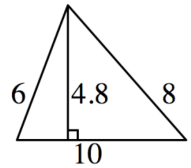
In a *triangle* with side lengths a, b, and c, and altitude height h to side b:

$P=a+b+c$
$A=\frac{1}{2}bh=\frac{bh}{2}$



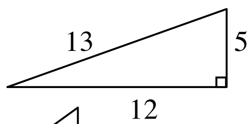
example:



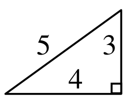
$P = a + b + c$
$= 6 + 8 + 10$
$= 24$ units
$A = \frac{1}{2}bh = \frac{1}{2}(10)(4.8) = 24$ square units

Problems 1-8: Find P and A:

1. Rectangle with sides 5 and 10.
2. Rectangle with sides 1.5 and 4.
3. Square with sides 3 miles.
4. Square with sides $\frac{3}{4}$ yards.
5. Parallelogram with sides 36 and 24, and height 10 (on side 36).
6. Parallelogram, all sides 12, altitude 6.
7. Triangle with sides 5, 12, and 13. Side 5 is the altitude on side 12.

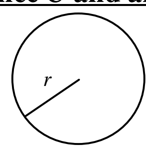


8. Triangle shown:



B. <u>Formulas for circumference C and area A of a circle</u>:

A circle with radius r (and diameter $d = 2r$) has a distance around (circumference) $C = \pi d = 2\pi r$ (If a piece of wire is bent into a circular shape, the circumference is the length of the wire.)



example: A circle with radius $r = 70$ has
$d = 2r = 140$ and exact circumference
$C = 2\pi r = 2 \bullet \pi \bullet 70 = 140\pi$ units
If π is approximated by $\frac{22}{7}$,
$C = 140\pi \approx 140\left(\frac{22}{7}\right) \approx 440$ units (approx.)
If π is approximated by 3.1,
$C \approx 140(3.1) = 434$ units
The area of a circle is $A = \pi r^2$
example: If $r = 8$, exact area is
$A = \pi r^2 = \pi \bullet 8^2 = 64\pi$ square units

Problems 9-11: Find the exact C and A for a circle with:

9. radius $r = 5$ units
10. $r = 10$ feet
11. diameter $d = 4$km

Problems 12-14: A circle has area 49π:

12. What is its radius length?
13. What is the diameter?
14. Find its circumference.

Problems 15-16: A parallelogram has area 48 and two sides each of length 12:

15. How long is the altitude to those sides?
16. How long are each of the other two sides?
17. How many times the P and A of a 3cm square are the P and A of a square with sides all 6 cm?
18. A rectangle has area 24 and one side 6. Find the perimeter.

Problems 19-20: A square has perimeter 30:

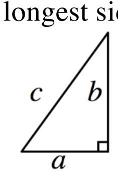
19. How long is each side?
20. What is its area?
21. A triangle has base and height each 7. What is its area?

C. <u>Pythagorean theorem</u>:

In any triangle with a $90°$ (right) angle, the sum of the squares of the legs equals the square of hypotenuse.

(The legs are the two shorter sides; the hypotenuse is the longest side.)

If the legs have lengths a and b, and c is the hypotenuse length, then $a^2 + b^2 = c^2$.



In words: " In a right triangle, leg squared plus leg squared equals hypotenuse squared."

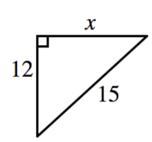
example: A right triangle has hypotenuse 5 and one leg 3. Find the other leg. Since $leg^2 + leg^2 = hyp^2$,
$3^2 + x^2 = 5^2$
$9 + x^2 = 25$
$x^2 = 25 - 9 = 16$
$x = \sqrt{16} = 4$

Problems 22-24: Find the length of the third side of the right triangle:

22. one leg: 15, hypotenuse: 17
23. hypotenuse: 10, one leg: 8
24. legs: 5 and 12

Problems 25-26: Find x:

25.



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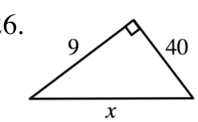

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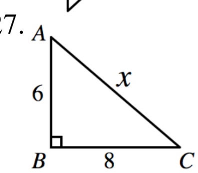
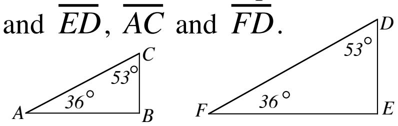


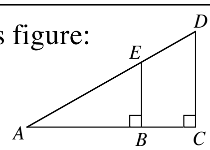
28. In right ΔRST with right angle R, $SR = 11$ and $TS = 61$. Find RT. (Draw and label a triangle to solve.)
29. Would a triangle with sides 7, 11, and 13 be a right triangle? Why or why not?

Similar triangles are triangles which are the same shape. If two angles of one triangle are equal respectively to two angles of another triangle, then the triangles are similar.

example: ΔABC and ΔFED are similar: The pairs of sides which correspond are $\overline{AB}$ and $\overline{FE}$, $\overline{BC}$ and $\overline{ED}$, $\overline{AC}$ and $\overline{FD}$.

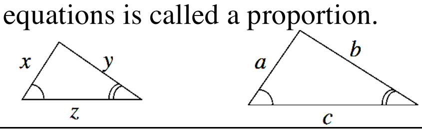


Problems 30-32: Use this figure:

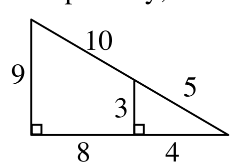


30. Find and name two similar triangles.
31. Draw the triangles separately and label them.
32. List the three pairs of corresponding sides.

If two triangles are similar, any two corresponding sides have the same ratio (fraction value): *example:* the ratio a to x, or $\frac{a}{x}$, is the same as $\frac{b}{y}$ and $\frac{c}{z}$. Thus $\frac{a}{x} = \frac{b}{y}$, $\frac{a}{x} = \frac{c}{z}$, and $\frac{b}{y} = \frac{c}{z}$. Each of these equations is called a proportion.

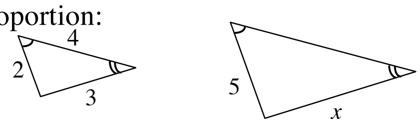


33. Draw the similar triangles separately, label them, and write proportions for the corresponding sides.



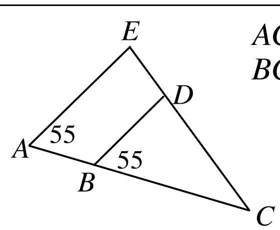
Problems 34-37: Solve for x:

example: Find x by writing and solving a proportion:



$\frac{2}{5} = \frac{3}{x}$, so cross multiply and get $2x = 15$ or $x = 7\frac{1}{2}$

34. $AC = 10$; $EC = 7$
$BC = 4$; $DC = x$



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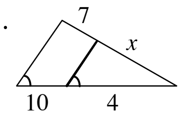

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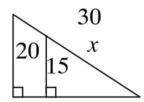

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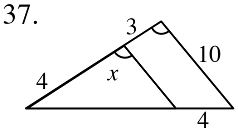
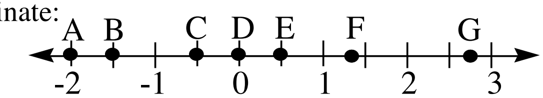


D. <u>Graphing on the number line</u>:

Problems 38-45: Name the point with given coordinate:



38. 0	42. -1.5
39. $\frac{1}{2}$	43. 2.75
40. $-\frac{1}{2}$	44. $-\frac{3}{2}$
41. $\frac{4}{3}$	45. $1.\overline{3}$

Problems 46-51: On the number line above, what is the distance between the listed points? (Remember that distance is always positive.)

46. D and G	49. B and C
47. A and D	50. B and E
48. A and F	51. F and G

Problems 52-55: On the number line, find the distance from:

52. –7 to –4	54. –4 to 7
53. –7 to 4	55. 4 to 7

Problems 56-59: Draw a sketch to help find the coordinate of the point…:

56. Halfway between points with coordinates 4 and 14.
57. Midway between –5 and –1.
58. Which is the midpoint of the segment joining –8 and 4.
59. On the number line the same distance from –6 as it is from 10.

E. <u>Coordinate plane graphing</u>:

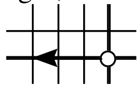
To locate a point on the plane, an ordered pair of numbers is used, written in the form (x, y).

Problems 60-63: Identify coordinates x and y in each ordered pair:

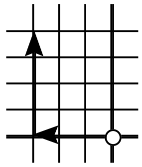
60. $(3,0)$	62. $(5,-2)$
61. $(-2,5)$	63. $(0,3)$

To plot a point, start at the origin and make the moves, first in the x-direction (horizontal) and then the y-direction (vertical) indicated by the ordered pair.

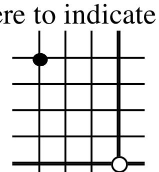
example: (-3, 4)
Start at the origin, move left 3 (since $x = -3$),



then (from there), up 4 (since $y = 4$),



put a dot there to indicate the point (-3, 4)

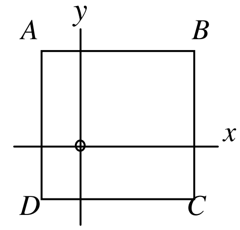


64. On graph paper, join these points in order: (–3,–2), (1,–4), (3, 0), (2, 3), (–1, 2), (3, 0), (–3, –2), (–1, 2), (1,–4).

65. Two of the lines drawn in problem 64 cross each other. What are the coordinates of the crossing point?

66. In what quadrant is the point (a,b) if $a > 0$ and $b < 0$?

Problems 67-69: $ABCD$ is a square, with $C(5,-2)$ and $D(-1,-2)$. Find:



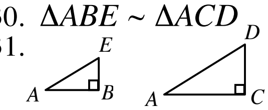
67. the length of each side.
68. the coordinates of A.
69. the coordinates of the midpoint of $\overline{DC}$.

Problems 70-72: Given $A(0,5)$, $B(12,0)$:

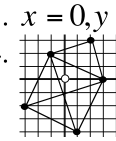
70. Sketch a graph. Draw $\overline{AB}$. Find its length.
71. Find the midpoint of $\overline{AB}$ and label it C. Find the coordinates of C.
72. What is the area of the triangle formed by A, B, and the origin?

Answers:

1. 30 *units*, 50 $units^2$ ($units^2$ means square units)
2. 11 *units*, 6 $units^2$
3. 12 *miles*, 9 $miles^2$
4. 3 *yards*, $\frac{9}{16}$ $yards^2$
5. 120 *un.*, 360 $un.^2$
6. 48 *un.*, 72 $un.^2$
7. 30 *un.*, 30 $un.^2$
8. 12 *un.*, 6 $un.^2$
9. 10π *un.*, 25π $un.^2$
10. 20π *ft.*, 100π $ft.^2$
11. 4π *km*, 4π km^2
12. 7
13. 14
14. 14π
15. 4
16. Cannot tell
17. P is 2 times, A is 4 times
18. 20
19. $7\frac{1}{2}$
20. $\frac{225}{4}$
21. $24\frac{1}{2}$
22. 8
23. 6
24. 13
25. 9
26. 41
27. 10
28. 60
29. No, because $7^2 + 11^2 \neq 13^2$
30. $\Delta ABE \sim \Delta ACD$
31.



32. $\overline{AB}, \overline{AC}$; $\overline{AE}, \overline{AD}$; $\overline{BE}, \overline{CD}$
33. $\frac{3}{9} = \frac{5}{15} = \frac{4}{12}$
34. $\frac{14}{5}$ or $2\frac{4}{5}$
35. $2\frac{4}{5}$ or $\frac{14}{5}$
36. $\frac{45}{2}$
37. $\frac{40}{7}$
38. D
39. E
40. C
41. F
42. B
43. G
44. B
45. F
46. 2.75
47. 2
48. $3\frac{1}{3}$
49. 1
50. 2
51. $\frac{17}{12}$
52. 3
53. 11
54. 11
55. 3
56. 9
57. –3
58. –2
59. 2
60. $x = 3, y = 0$
61. $x = -2, y = 5$
62. $x = 5, y = -2$
63. $x = 0, y = 3$
64.



65. (0,–1)
66. IV
67. 6
68. $(-1,4)$
69. $(2,-2)$
70. 13
71. $(6,\frac{5}{2})$
72. 30