Level 1

ALGEBRA READINESS DIAGNOSTIC TEST PRACTICE

Directions: Study the examples, work the problems, then check your answers at the end of each topic. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands the topic.

TOPIC 1: INTEGERS

A. What is an integer?

Any natural number (1, 2, 3, 4, 5,…), its opposite (-1,-2, -3, -4, -5,…), or zero (0). (Integers are useful for problems involving "below normal," debts, "below sea level," etc.)

Problems 1-10: Identify each number as an integer (I) or not an integer (NI):

1. 367
2. -4.4
3. $2\frac{1}{2}$
4. –1010
5. $\sqrt{100}$
6. 0
7. $-\frac{2}{3}$
8. 0.027
9. $\frac{1}{2}$
10. 2^3

Problems 11-14: Write the opposite of each integer:

11. 42
12. -3
13. 0
14. -4^3

Problems 15-19: Choose the greater:

15. 5, –10
16. 5,–5
17. 5, 0
18. –5, 0
19. –5,–10

20. What is the result of adding an integer and its opposite?
21. What number is its own opposite?

B. Absolute Value:

Absolute value is used for finding distance, explaining addition of integers, etc.
The absolute value of a positive number or zero is itself. The absolute value of a negative number is its opposite.

Problems 22-26: Choose the integer with the greater absolute value:

22. 4 or –3
23. –4 or 3
24. 3 or –3
25. 3 or 0
26. – 3 or 0

C. Adding, subtracting, multiplying and dividing integers:

To add two integers:

Both positive: add as natural numbers:
example: Add 4 and 3: $4 + 3 = 7$

Both negative: add as though positive; make the result negative:
example: Add –4 and –3:
Treat as positive and add: $4 + 3 = 7$.
The answer is –7 because it must be negative.

One positive, one negative: treat each as positive, subtract, make the answer sign of the one with the greater absolute value:
example: Add –4 and 3: $4 - 3 = 1$; the answer is –1 because –4 has the greater absolute value.
example: Add 4 and –3: $4 - 3 = 1$; the answer is 1 because 4 has the greater absolute value.

Problems 27-33: Add the two integers:

27. 4 and –3 (This means $(4)+(-3)$)
28. 4 and 3
29. –4 and –3
30. 4 and 0
31. –4 and 3
32. 16 and –7
33. –3 and 0

To subtract two integers: add the opposite of the one to be subtracted:
example: 3 subtract –4, or $(3)-(-4)$: The opposite of –4 is 4, so we add 4 (rather than subtract –4).
We change the problem from $(3)-(-4)$ to $(3)+(4)$, which we know how to do:
$(3)+(4) = 3 + 4 = 7$
example: $(-4)-(3)$: Add the opposite of 3, namely –3: $(-4)-(3) = (-4)+(-3) = -7$
example: $(4)-(3) = (4)+(-3) = 1$
example: $-5 - 8 = (-5)-(8) = (-5)+(-8) = -13$

Problems 34-43: Calculate:

34. $(12)-(3) =$
35. $-12 - 3 =$ (Hint: this means $-12+(-3)$)
36. $-12 - (-3) =$
37. $3 - 12 =$
38. $-3 - 12 =$
39. $(-7)-(-7) =$
40. $0 - 3 =$
41. $0 + 4 =$
42. $-12 + 3 =$
43. $(-3)+(-12) =$

To multiply two integers:

1st integer ×	2nd integer =	Answer
+	+	+
−	+	−
+	−	−
−	−	+

Both positive: multiply as two natural numbers.
example: $(3)\times(4)=3\times 4=12$

Both negative: multiply as if positive; and make the answer positive. and remember, two negatives make a positive. When multiplying two negative numbers, you always get a positive answer.
example: $(-3)(-4)$ so $3\times 4=12$; make it positive, and the answer is 12.

One positive, one negative: When multiplying a negative number and a positive number, the answer is always negative.
example: $(3)(-4)$ so $3\times 4=12$; make the answer negative; answer –12.

Problems 44-55: Multiply:

44. $3\times(-4)=$	50. $(-4)\bullet 0=$
45. $(3)\bullet(-4)=$	51. $0^2=$
46. $(3)(-4)=$	52. $(-3)^2=$
47. $3(-4)=$	53. $(4)^2=$
48. $(-3)(-4)=$	54. $(-3)\bullet 4=$
49. $-3(-4)=$	55. $3\bullet 4=$

Reciprocals are used for dividing. Every integer except zero has a reciprocal. The reciprocal is the number that multiplies the integer to give 1.

example: $6\bullet\frac{1}{6}=1$, so the reciprocal of 6 is $\frac{1}{6}$. (And the reciprocal of $\frac{1}{6}$ is 6.)
example: $(-4)\left(-\frac{1}{4}\right)=1$, so the reciprocal of –4 is $-\frac{1}{4}$.

Problems 56-59: Find the reciprocal:

56. –5	57. 1	58. 10	59. –1

60. What number is its own reciprocal? (Can you find "more than one"?)
61. Using the reciprocal definition, explain why there is no reciprocal of zero.

To divide two integers: multiply by the reciprocal of the one to be divided by:
example: 20 divided by $-5=20\div(-5)$.
The reciprocal of –5 is $-\frac{1}{5}$ so we multiply by $-\frac{1}{5}$: $20\div(-5)=20\times\left(-\frac{1}{5}\right)=$
$\frac{20}{1}\times\left(-\frac{1}{5}\right)=-\frac{20\bullet 1}{1\bullet 5}=-\frac{20}{5}=-4$
example: $\frac{-5}{20}=-5\div 20=-5\bullet\frac{1}{20}=-\frac{1}{4}$
example: $\frac{-3}{-6}=-3\div(-6)=-3\bullet\left(-\frac{1}{6}\right)=\frac{3}{6}=\frac{1}{2}$
(Note negative times negative is positive.)

example: $\frac{0}{3}=0\div 3=0\bullet\frac{1}{3}=0$

Problems 62-67: Calculate:

62. $(-14)\div(-2)=$	65. $\frac{-15}{3}=$
63. $2\div 3=$	66. $\frac{-5}{0}=$(careful)*
64. $3\div 2=$	67. $\frac{0}{7}=$

* Problem 61 says $\frac{1}{0}$ has no value (you cannot divide by zero).

68. From the rule for division, why is it impossible to divide by zero?

To "sum" it all up:
Positive+positive = larger positive
Negative+negative = more negative
Positive+negative = in between both

Positive × positive = positive
Negative × negative = positive
Positive × negative = negative

To subtract add the opposite.
To divide, multiply by the reciprocal.

69. Given the statement "Two negatives make a positive." Provide an example of a situation where the statement would be true and another when it would be false.
70. Write "18 divided by 30" in three ways: using ÷, $\overline{)\ }$, and using a fraction bar —.

Problems 71- 80: Calculate:

71. $4-10+3-2=$	76. $-2[(-6)(8)+9]=$
72. $4-(10+3-2)=$	77. $5+(3-7)=$
73. $4+3-(10-2)=$	78. $5-(3-7)=$
74. $6(8-3)=$	79. $5-3+7=$
75. $(-6)(8)+9=$	80. $-1+2-3+4=$

81. What is the meaning of "sum", "product", quotient", and "difference"?

D. Factoring:
If a number is the product of two (or more) integers, then the integers are factors of the number.

example: $40=4\times 10$, 2×20, 1×40, and 8×5. So 1, 2, 4, 5, 8, 10, 20, 40 are all factors of 40. (So are all their negatives.)

Problems 82-86: Find all positive factors of:

82. 10	83. 7	84. 24	85. 9	86. 1

If a positive integer has exactly two positive factors, it is a prime number. Prime numbers are used to find the greatest common factor (GCF) and least common multiple (LCM), which are used to reduce fractions and find common denominators, which in turn are often needed for adding and subtracting fractions.

example: The only positive factors of 7 are 1 and 7, so 7 is a prime number.
example: 6 is not prime, as it has 4 positive factors: 1, 2, 3, 6.

87. From the prime number definition, why is 1 *not* a prime?
88. Write the 25 prime numbers from 1 to 100.

Every positive integer has one way it can be factored into primes, called its prime factorization.

example: Find the prime factorization (PF) of 30: $30 = 3 \times 10 = 3 \times 2 \times 5$, so the PF of 30 is $2 \bullet 3 \bullet 5$.
example: $72 = 2 \bullet 2 \bullet 2 \bullet 3 \bullet 3 = 2^3 \bullet 3^2$, the PF. (The PF can be found by making a "factor tree.")

Problems 89-91: Find the PF:

89. 36	90. 10	91. 7

Greatest common factor (GCF) and least common multiple (LCM). If you need to review GCF or LCM, see the worksheet in this series: "Topic 2: Fractions".

Problems 92-95: Find the GCF and the LCM of:

92. 4 and 6	93. 4 and 7
94. 4 and 8	95. 3 and 5

E. Word problems:

96. The temperature goes from $-14°$ to $28°$C. How many degrees Celsius does it change?
97. $28-(-14)=$
98. Derek owes \$43, has \$95, so "is worth"…?
99. If you hike in Death Valley from 282 feet below sea level to 1000 feet above sea level, how many feet of elevation have you gained?
100. $1000-(-282)=$
101. A hike from 243 feet below sea level (FBSL) to 85 FBSL means a gain in elevation of how many feet?
102. $-85-(-243)=$
103. What number added to –14 gives –24?
104. What does "an integral number" mean?
105. Jim wrote a check for \$318. His balance is then \$2126. What was the balance before he wrote the check?
106. What number multiplied by 6 gives –18?
107. If you hike downhill and lose 1700 feet of elevation and end at 3985 feet above sea level (FASL), what was your starting elevation?
108. Anne was 38 miles south of her home. She drove 56 miles north. How far from home was she at that time and in what direction?
109. 5 subtracted from what number gives –12?
110. What number minus negative four gives ten?

Answers:

1. I
2. NI
3. NI
4. I
5. I
6. I
7. NI
8. NI
9. NI
10. I
11. –42
12. 3
13. 0
14. 64
15. 5
16. 5
17. 5
18. 0
19. –5
20. zero
21. zero
22. 4
23. –4
24. both same
25. 3
26. –3
27. 1
28. 7
29. –7
30. 4
31. –1
32. 9
33. –3
34. 9
35. –15
36. –9
37. –9
38. –15
39. 0
40. –3
41. 4
42. –9
43. –15
44. –12
45. –12
46. –12
47. –12
48. 12
49. 12
50. 0
51. 0
52. 9
53. 16
54. –12
55. 12
56. $-\frac{1}{5}$
57. 1
58. $\frac{1}{10}$
59. –1
60. 1; also –1
61. no number times $0 = 1$
62. 7
63. $\frac{2}{3}$
64. $\frac{3}{2}$

65. –5
66. no value (not defined)
67. 0
68. zero has no reciprocal
69. true if ×, false if +.
70. $18 \div 30$, $30\overline{)18}$, 18/30
71. –5
72. –7
73. –1
74. 30
75. –39
76. 78
77. 1
78. 9
79. 9
80. 2
81. +, ×, ÷, −
82. 1, 2, 5, 10
83. 1, 7
84. 1, 2, 3, 4, 6, 8, 12, 24
85. 1, 3, 9
86. 1
87. 1 has one factor
88. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
89. $2^2 \bullet 3^2$
90. $2 \bullet 5$
91. 7
92. 2, 12
93. 1, 28
94. 4, 8
95. 1, 15
96. 42
97. 42
98. \$52
99. 1282
100. 1282
101. 158
102. 158
103. –10
104. an integer
105. \$2444
106. –3
107. 5685 FASL
108. 18 mi. N
109. –7
110. 6

TOPIC 2: FRACTIONS

A. <u>Greatest Common Factor (GCF)</u>:
The GCF of two integers is used to simplify (reduce, rename) a fraction to an equivalent fraction. A <u>factor</u> is an integer <u>multiplier</u>. A <u>prime number</u> is a positive whole number with exactly two positive factors.

> *example:* the prime factorization of 18 is
> $2 \bullet 3 \bullet 3$, or $2 \bullet 3^2$.

Problems 1-2: Find the prime factorization:

1. 24
2. 42

> *example:* Find the factors of 42.
> Factor into primes: $42 = 2 \bullet 3 \bullet 7$
> 1 is always a factor
> 2 is a prime factor
> 3 is a prime factor
> 4 is a prime factor
> 7 is a prime factor
> $2 \bullet 3 = 6$ is a factor
> $2 \bullet 7 = 14$ is a factor
> $3 \bullet 7 = 21$ is a factor
> $2 \bullet 3 \bullet 7 = 42$ is a factor
> Thus 42 has 8 factors.

Problems 3-4: Find all positive factors:

3. 18
4. 24

To find the GCF:

> *example:* Looking at the factors of 42 and 24, we see that the common factors of both are 1, 2, 3, and 6, of which the greatest it 6; so: the GCF of 42 and 24 is 6. (Notice that "common factor" means "shared factor.")

Problems 5-7: Find the GCF of:

5. 18 and 36
6. 27 and 36
7. 8 and 15

B. <u>Simplifying fractions</u>:

> *example:* Reduce $\frac{27}{36}$:
> $\frac{27}{36} = \frac{9 \bullet 3}{9 \bullet 4} = \frac{9}{9} \bullet \frac{3}{4} = 1 \bullet \frac{3}{4} = \frac{3}{4}$
> (Note that you must be able to find a common factor, in this case 9, in both the top and bottom in order to reduce.)

Problems 8-13: Reduce:

8. $\frac{13}{52} =$
9. $\frac{26}{65} =$
10. $\frac{3+6}{3+9} =$
11. $\frac{16}{64} =$
12. $\frac{24}{42} =$
13. $\frac{24}{18} =$

C. <u>Equivalent Fractions</u>:

> *example:* $\frac{3}{4}$ is equivalent to how many eighths?
> $\left(\frac{3}{4} = \frac{}{8}\right)$
> $\frac{3}{4} = 1 \bullet \frac{3}{4} = \frac{2}{2} \bullet \frac{3}{4} = \frac{2 \bullet 3}{2 \bullet 4} = \frac{6}{8}$

Problems 14-17: Complete:

14. $\frac{4}{9} = \frac{}{72}$
15. $\frac{3}{5}$ is how many twentieths?
16. $\frac{56}{100} = \frac{}{50}$
17. How many halves are in 3? (Hint: think $3 = \frac{3}{1} = \frac{}{2}$)

D. <u>Ratio</u>:
If the ratio of boys to girls in a class is 2 to 3, it means that for every 2 boys, there are 3 girls. A

ratio is like a fraction: think of the ratio 2 to 3 as the fraction $\frac{2}{3}$.

example: If the class had 12 boys, how many girls are there? Write the fraction ratio: $\frac{number\ of\ boys}{number\ of\ girls} = \frac{2}{3} = \frac{12}{}$

Complete the equivalent fraction: $\frac{2}{3} = \frac{2 \bullet 6}{3 \bullet 6} = \frac{12}{18}$

So there are 18 girls.

18. If the class had 21 girls and the ratio of boys to girls was 2 to 3, how many boys would be in the class?
19. If the ratio of X to Y is 4 to 3, and there are 462 Y's, how many X's are there?
20. If the ratio of games won to games played is 6 to 7 and 18 games were won, how many games were played?

E. Least common multiple (LCM):

The LCM of two or more integers is used to find the lowest common denominator of fractions in order to add or subtract them.

To find the LCM:
example: Find the LCM of 27 and 36.

First factor into primes:

$27 = 3^3$

$36 = 2^2 \bullet 3^2$

Make the LCM by taking each prime factor to its greatest power:

$\text{LCM} = 2^2 \bullet 3^3 = 4 \bullet 27 = 108$

Problems 21-25: Find the LCM:

21. 6 and 15
22. 4 and 8
23. 3 and 5
24. 8 and 12
25. 8, 12, and 15

F. Lowest common denominator (LCD):

To find LCD fractions for two or more given fractions:
example: Given $\frac{5}{6}$ and $\frac{8}{15}$

First find LCM of 6 and 15:

$6 = 2 \bullet 3$

$15 = 3 \bullet 5$

$\text{LCM} = 2 \bullet 3 \bullet 5 = 30 = \text{LCD}$

So $\frac{5}{6} = \frac{25}{30}$ and $\frac{8}{15} = \frac{16}{30}$

Problems 26-32: Find equivalent fractions with the LCD:

26. $\frac{2}{3}$ and $\frac{2}{9}$
27. $\frac{3}{8}$ and $\frac{7}{12}$
28. $\frac{4}{5}$ and $\frac{2}{3}$
29. $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$
30. $\frac{7}{8}$ and $\frac{5}{8}$
31. Which is larger, $\frac{5}{7}$ or $\frac{3}{4}$? (Hint: find and compare LCD fractions)
32. Which is larger, $\frac{3}{8}$ or $\frac{1}{3}$?

G. Adding and subtracting fractions:

If denominators are the same, combine the numerators:
example: $\frac{7}{10} - \frac{1}{10} = \frac{7-1}{10} = \frac{6}{10} = \frac{3}{5}$

Problems 33-37: Find the sum or difference (reduce if possible):

33. $\frac{4}{7} + \frac{2}{7} =$
34. $\frac{5}{6} + \frac{1}{6} =$
35. $\frac{7}{8} - \frac{5}{8} =$
36. $3 + \frac{1}{2} =$
37. $1 - \frac{2}{3} =$

If the denominators are different, first find equivalent fractions with common denominators (preferably the LCD):
example: $\frac{4}{5} + \frac{2}{3} = \frac{12}{15} + \frac{10}{15} = \frac{22}{15} = 1\frac{7}{15}$
example: $\frac{1}{2} - \frac{2}{3} = \frac{3}{6} - \frac{4}{6} = \frac{3-4}{6} = \frac{-1}{6}$

Problems 38-43: Calculate:

38. $\frac{3}{5} - \frac{2}{3} =$
39. $\frac{5}{8} + \frac{1}{4} =$
40. $\frac{5}{2} + \frac{5}{4} =$
41. $2\frac{3}{4} + 5\frac{7}{8} =$
42. $\left(3\frac{1}{4} - \frac{3}{4}\right) + \frac{1}{2} =$
43. $4\frac{1}{3} - \left(3\frac{1}{2} - 3\right) =$

H. Multiplying and dividing fractions:

To multiply fractions, multiply the tops, multiply the bottoms, and reduce if possible:
example: $\frac{3}{4} \bullet \frac{2}{5} = \frac{3 \bullet 2}{4 \bullet 5} = \frac{6}{20} = \frac{3}{10}$

Problems 44-52: Calculate:

44. $\frac{2}{3} \bullet \frac{3}{8} =$
45. $\frac{1}{2} \bullet \frac{2}{3} =$
46. $\frac{4}{5} \times 5 =$
47. $\left(\frac{3}{4}\right)^2 =$
48. $\left(\frac{3}{2}\right)^2 =$
49. $\left(2\frac{1}{2}\right)^2 =$
50. $\frac{4}{5} \bullet 30 =$
51. $8 \bullet \frac{3}{4} =$
52. $\frac{15}{21} \times \frac{14}{25} =$

Divide fractions by making a compound fraction and then multiply the top and bottom (of the larger fraction) by the lowest common denominator (LCD) of both.

example: $\frac{3}{4} \div \frac{2}{3} = \frac{\frac{3}{4}}{\frac{2}{3}}$

The LCD is 12, so multiply by 12: $\frac{\frac{3}{4} \bullet 12}{\frac{2}{3} \bullet 12} = \frac{9}{8}$

example: $\dfrac{7}{\frac{2}{3}-\frac{1}{2}} = \dfrac{7\bullet 6}{\left(\frac{2}{3}-\frac{1}{2}\right)\bullet 6}$

$= \dfrac{42}{\frac{2}{3}\bullet 6-\frac{1}{2}\bullet 6} = \dfrac{42}{4-3} = \dfrac{42}{1} = 42$

Problems 53-62: Calculate:

53. $\frac{3}{2} \div \frac{1}{4} =$

54. $11\frac{3}{8} \div \frac{3}{4} =$

55. $\frac{3}{4} \div 2 =$

56. $\dfrac{\frac{3}{4}}{\frac{2}{3}} =$

57. $\dfrac{1+\frac{1}{2}}{1-\frac{3}{4}} =$

58. $\dfrac{2}{\frac{3}{4}} =$

59. $\dfrac{\frac{2}{3}}{4} =$

60. $\frac{4}{5} \div 5 =$

61. $\frac{3}{8} \div 3 =$

62. $\dfrac{2\frac{1}{3}-\frac{1}{3}}{3\frac{2}{3}+1\frac{1}{2}} =$

I. <u>Comparing fractions</u>:

example: Arrange small to large: $\frac{7}{9}$, $\frac{5}{7}$, and $\frac{3}{4}$

LCD is $2^2 \bullet 3^2 \bullet 7 = 252$

$\frac{7}{9} = \frac{7\bullet 28}{9\bullet 28} = \frac{196}{252}$

$\frac{5}{7} = \frac{5\bullet 36}{7\bullet 36} = \frac{180}{252}$

$\frac{3}{4} = \frac{3\bullet 63}{4\bullet 63} = \frac{189}{252}$

So the order is $\frac{5}{7}$, $\frac{3}{4}$, $\frac{7}{9}$

Fractions can also be compared by writing in decimal from and comparing the decimals.

Problems 63-65: Arrange small to large:

63. $\frac{15}{8}$, $\frac{11}{6}$

64. $\frac{7}{8}$, $\frac{5}{6}$, $\frac{11}{12}$

65. $\frac{2}{3}$, $\frac{7}{12}$, $\frac{5}{6}$, $\frac{25}{36}$

<u>Word Problems</u>:

66. How many 2's are in 8?

67. How many $\frac{1}{2}$'s are in 8?

68. Three fourths is equal to how many twelfths?

69. What is $\frac{3}{4}$ of a dozen?

70. Joe and Mae are decorating the gym for a dance. Joe has done $\frac{1}{3}$ of the work and Mae has done $\frac{2}{5}$. What fraction of the work still must be done?

71. The ratio of winning tickets to tickets sold is 2 to 5. If 3,500,000 are sold, how many tickets are winners?

72. An $11\frac{3}{8}$-inch wide board can be cut into how many strips of width $\frac{5}{8}$ inch, if each cut takes $\frac{1}{8}$ inch of the width? (Must the answer be a whole number?)

Problems 73-76: Inga and Lee each work for $4.60 per hour:

73. If Inga works $3\frac{1}{2}$ hours, what will her pay be?

74. If Lee works $2\frac{3}{4}$ hours, what will he be paid?

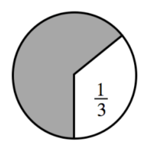
75. Together, what is the total time they work?

76. What is their total pay?

<u>Visual Problems</u>:

Problems 77-80: What fraction of the figure is <u>shaded</u>?

77.



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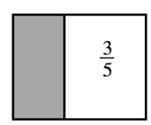

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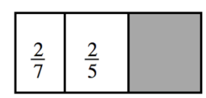

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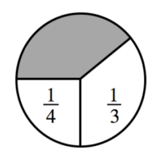
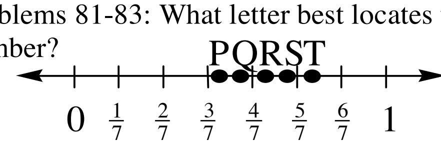


Problems 81-83: What letter best locates the given number?



81. $\frac{5}{9}$

82. $\frac{3}{4}$

83. $\frac{2}{3}$

<u>Answers</u>:

1. $2^3 \bullet 3$
2. $2\bullet 3\bullet 7$
3. 1, 2, 3, 6. 9, 18
4. 1, 2, 3, 4, 6, 8, 12, 24
5. 18
6. 9
7. 1
8. $\frac{1}{4}$
9. $\frac{2}{5}$
10. $\frac{3}{4}$
11. $\frac{1}{4}$
12. $\frac{4}{7}$
13. $\frac{4}{3}$
14. 32
15. 12
16. 28
17. 6
18. 14
19. 616
20. 21
21. 30
22. 8
23. 15
24. 24
25. 120
26. $\frac{6}{9}$, $\frac{2}{9}$
27. $\frac{9}{24}$, $\frac{14}{24}$
28. $\frac{12}{15}$, $\frac{10}{15}$
29. $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$
30. $\frac{7}{8}$, $\frac{5}{8}$
31. $\frac{3}{4}$ (because $\frac{20}{28} < \frac{21}{28}$)
32. $\frac{3}{8}$ (because $\frac{9}{24} > \frac{8}{24}$)